1. $\int 4x^5 \, dx$

$$\int 4x^5 \, dx = 4 \int x^5 \, dx$$
$$= \frac{4}{5+1} x^{5+1}$$
$$= \frac{4}{6} x^6 + c$$
$$= \frac{2}{3} x^6 + c$$

2. $\int 2\sqrt[3]{x} \, dx$

$$\int 2\sqrt[3]{x} \, dx = 2 \int x^{\frac{1}{3}}$$
$$= \frac{2}{\frac{1}{3}+1} x^{\frac{1}{3}+1} + c$$
$$= \frac{2}{\frac{4}{3}} x^{\frac{4}{3}} + c$$
$$= \frac{6}{3} x^{\frac{4}{3}} + c$$
$$= \frac{3}{2} x^3 \sqrt{x} + c$$

3. $\int x^4 \sqrt{x} \, dx$

$$\int x^4 \sqrt{x} \, dx = \int x^4 x^{\frac{1}{2}} \, dx = \int x^{\frac{9}{2}} \, dx$$
$$= \frac{1}{\frac{9}{2}+1} x^{\frac{9}{2}+1} + c$$
$$= \frac{1}{\frac{11}{2}} x^{\frac{11}{2}} + c$$
$$= \frac{2}{11} x^{\frac{11}{2}} + c$$

4. $\int (x+3)^2 \, dx$

$$\int (x+3)^2 \, dx = \int (x^2 + 6x + 9) \, dx$$
$$= \frac{1}{3} x^3 + 3x^2 + 9x + c$$

5. $\int \frac{5x^3(x-2)}{10\sqrt{x}}\, dx$

$$\int \frac{5x^3(x-2)}{10\sqrt{x}}\, dx = \int \frac{5x^4-10x^3}{10x^{\frac{1}{2}}}\, dx$$

$$= \int \frac{5x^4}{10x^{\frac{1}{2}}} - \frac{10x^3}{10x^{\frac{1}{2}}}\, dx$$

$$= \int \frac{1}{2}x^{\left(4-\frac{1}{2}\right)} - x^{\left(3-\frac{1}{2}\right)}\, dx$$

$$= \int \frac{1}{2}x^{\frac{7}{2}} - x^{\frac{5}{2}}\, dx$$

$$= \int \frac{1}{2}x^{\frac{7}{2}}dx - \int x^{\frac{5}{2}}\, dx$$

$$= \frac{\frac{1}{2}}{\frac{7}{2}+1}x^{\left(\frac{7}{2}+1\right)} - \frac{1}{\frac{5}{2}+1}x^{\left(\frac{5}{2}+1\right)} + c$$

$$= \frac{\frac{1}{2}}{\frac{9}{2}}x^{\frac{9}{2}} - \frac{1}{\frac{7}{2}}x^{\frac{7}{2}} + c$$

$$= \frac{1}{9}x^{\frac{9}{2}} - \frac{2}{7}x^{\frac{7}{2}} + c$$